

JEE-MAIN EXAMINATION – JANUARY 2026

(HELD ON FRIDAY 23rd JANUARY 2026)

TIME : 9:00 AM TO 12:00 NOON

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let the domain of the function $f(x) = \log_3 \log_5 \log_7 (9x - x^2 - 13)$ be the interval (m, n) . Let the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ have eccentricity $\frac{n}{3}$ and the length of the latus rectum $\frac{8m}{3}$. Then $b^2 - a^2$ is equal to :

- (1) 5 (2) 11
(3) 9 (4) 7

Ans. (4)

Sol. $\log_5 (\log_7 (9x - x^2 - 13)) > 0$

$$\Rightarrow 9x - x^2 - 13 > 7$$

$$x^2 - 9x + 20 < 0 \Rightarrow 4 < x < 5$$

$$m = 4, n = 5$$

$$\Rightarrow e = \sqrt{1 + \frac{b^2}{a^2}} = \frac{5}{3} \Rightarrow \frac{b^2}{a^2} = \frac{25}{9} - 1 = \frac{16}{9}$$

$$\frac{b}{a} = \frac{4}{3}$$

$$\Rightarrow \frac{2b^2}{a} = \frac{8m}{3} \Rightarrow \frac{2b^2}{a} = \frac{32}{3}$$

$$\Rightarrow 2b^2 = \frac{32}{3} \times \frac{3b}{4} \Rightarrow b = 4, a = 3$$

$$b^2 - a^2 = 16 - 9 = 7$$

2. Let $f(x) = \int \frac{(2-x^2).e^x}{(\sqrt{1+x})(1-x)^{3/2}} dx$. If $f(0) = 0$, then $f\left(\frac{1}{2}\right)$ is equal to :

- (1) $\sqrt{3e} - 1$ (2) $\sqrt{2e} + 1$
(3) $\sqrt{2e} - 1$ (4) $\sqrt{3e} + 1$

Ans. (1)

Sol. $\int e^x \left(\frac{(1-x^2)+1}{\sqrt{1+x} \cdot (1-x)^{3/2}} \right) dx$

$$\int e^x \left(\frac{(1-x^2)}{\sqrt{1+x} \cdot (1-x)^{3/2}} + \frac{1}{\sqrt{1+x} \cdot (1-x)^{3/2}} \right) dx$$

$$\int e^x \left(\sqrt{\frac{1+x}{1-x}} + \frac{1}{\sqrt{1+x} \cdot (1-x)^{3/2}} \right) dx$$

$$= e^x \sqrt{\frac{1+x}{1-x}} + C$$

$$f(x) = e^x \sqrt{\frac{1+x}{1-x}} - 1$$

$$f\left(\frac{1}{2}\right) = \sqrt{3e} - 1$$

3. Let $\vec{a} = -\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} - \hat{j} - 3\hat{k}$, $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d} = \vec{c} \times \vec{a}$. Then $(\vec{a} - \vec{b}) \cdot \vec{d}$ is equal to :

- (1) 4 (2) -4
(3) -2 (4) 2

Ans. (3)

Sol. $\vec{d} = (\vec{a} \times \vec{b}) \times \vec{a}$

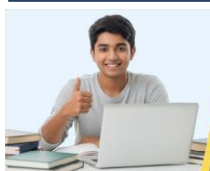
$$\vec{d} = (a^2)\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$$

$$\vec{d} = 6\vec{b} + 8\vec{a}$$

$$(\vec{a} - \vec{b}) \cdot \vec{d} = (\vec{a} - \vec{b}) \cdot (6\vec{b} + 8\vec{a})$$

$$= 8a^2 - 6b^2 - 2\vec{a} \cdot \vec{b}$$

$$= 48 - 66 + 16 = -2$$



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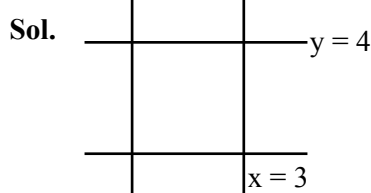
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4. A rectangle is formed by the lines $x = 0$, $y = 0$, $x = 3$ and $y = 4$. Let the line L be perpendicular to $3x + y + 6 = 0$ and divide the area of the rectangle into two equal parts. Then the distance of the point $\left(\frac{1}{2}, -5\right)$ from the line L is equal to :

- (1) $2\sqrt{5}$ (2) $3\sqrt{10}$
(3) $\sqrt{10}$ (4) $2\sqrt{10}$

Ans. (4)



Line is $y = \frac{x}{3} + C$

Line passes thru $\left(\frac{3}{2}, 2\right)$

$2 = \frac{1}{2} + C \Rightarrow C = \frac{3}{2}$

$y = \frac{x}{3} + \frac{3}{2}$

$\Rightarrow 6y = 2x + 9$

Line is $2x - 6y + 9 = 0$ &

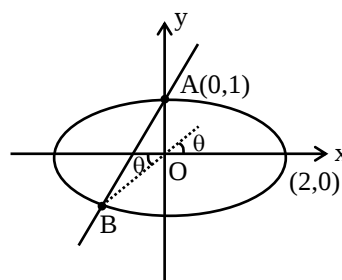
Dist = $\left| \frac{1+30+9}{\sqrt{40}} \right| = \sqrt{40} = 2\sqrt{10}$

5. Let the line $y - x = 1$ intersect the ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$ at the points A and B. Then the angle made by the line segment AB at the center of the ellipse is :

- (1) $\pi - \tan^{-1}\left(\frac{1}{4}\right)$ (2) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$
(3) $\frac{\pi}{2} + 2\tan^{-1}\left(\frac{1}{4}\right)$ (4) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$

Ans. (2)

Sol.



By solving line & equation of ellipse we get $x = 0$

& $x = -\frac{4}{3}$

$\therefore B\left(-\frac{4}{3}, -\frac{1}{3}\right)$

$m_{OB} = \tan \theta = \frac{1}{4}$

$\therefore \angle AOB = \frac{\pi}{2} + \theta = \frac{\pi}{2} + \tan^{-1} \frac{1}{4}$

6. Let $A = \{-2, -1, 0, 1, 2, 3, 4\}$. Let R be a relation on A defined by xRy if and only if $2x + y \leq 2$. Let l be the number of elements in R . Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations respectively. Then $l + m + n$ is equal to :

- (1) 32 (2) 34
(3) 33 (4) 35

Ans. (3)

Sol. $R\{(-2, a), (-1, b), (0, c), (1, d), (2, e)\}$

$a = \{-2, -1, 0, 1, 2, 3, 4\}; b = \{-2, -1, 0, 1, 2, 3, 4\}$

$c = \{-2, -1, 0, 1, 2\}; d = \{-2, -1, 0\}$

$e = \{-2\}$

$\therefore$ No. of elements in R

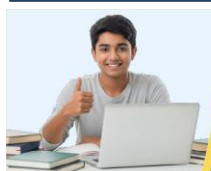
$= 7 + 7 + 5 + 3 + 1 = 23 = l$

Minimum number of element to be added to make it reflexive $= m = 4 \Rightarrow \{(1, 1), (2, 2), (3, 3), (4, 4)\}$
minimum number of element to be added to make it symmetric $= n = 6$

for 'n'

$\Rightarrow R = \{(3, -2), (4, -2), (2, -1), (2, 0), (3, -1), (4, -1)\}$

$\therefore l + m + n = 23 + 4 + 6 = 33$



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7. Let $y = y(x)$ be the solution of the differential equation

$$x^4 dy + (4x^3 y + 2 \sin x) dx = 0, x > 0, y\left(\frac{\pi}{2}\right) = 0.$$

Then $\pi^4 y\left(\frac{\pi}{3}\right)$ is equal to :

- (1) 81 (2) 92
(3) 64 (4) 72

Ans. (1)

Sol. $(x^4 dy + 4x^3 y dx) = -2 \sin x dx$

$$\Rightarrow \int d(x^4 y) = \int -2 \sin x dx$$

$$\Rightarrow x^4 y = 2 \cos x + c$$

$$\Rightarrow x^4 f(x) = 2 \cos x + c$$

$$\text{As } f\left(\frac{\pi}{2}\right) = 0$$

$$\text{So, } c = 0$$

$$\left(\frac{\pi}{3}\right)^4 f\left(\frac{\pi}{3}\right) = 2 \cos \frac{\pi}{3}$$

$$\pi^4 f\left(\frac{\pi}{3}\right) = 81$$

8. If α and β ($\alpha < \beta$) are the roots of the equation

$$(-2 + \sqrt{3}) (|\sqrt{x} - 3|) + (x - 6\sqrt{x}) + (9 - 2\sqrt{3}) = 0,$$

$x \geq 0$, then $\sqrt{\frac{\beta}{\alpha}} + \sqrt{\alpha\beta}$ is equal to :

- (1) 8 (2) 9
(3) 10 (4) 11

Ans. (3)

$$\text{Sol. } (x - 6\sqrt{x} + 9) - (2 - \sqrt{3}) |\sqrt{x} - 3| - 2\sqrt{3} = 0$$

$$\Rightarrow |\sqrt{x} - 3|^2 - (2 - \sqrt{3}) |\sqrt{x} - 3| - 2\sqrt{3} = 0$$

$$\Rightarrow |\sqrt{x} - 3| = 2 \text{ or } |\sqrt{x} - 3| = -\sqrt{3} \text{ (not possible)}$$

$$\Rightarrow \sqrt{x} = 1 \text{ or } 5$$

$$\Rightarrow x = 1 \text{ or } 25$$

$$\Rightarrow \alpha = 1 \text{ and } \beta = 25$$

Aliter:

$$\text{Let } x \geq 9, \text{ let } \sqrt{x} = t \Rightarrow t \geq 3$$

$$(\sqrt{3} - 2)(t - 3) + (t - 3)^2 - 2\sqrt{3} = 0$$

$$\text{Let } t - 3 = u$$

$$u^2 + (\sqrt{3} - 2)u - 2\sqrt{3} = 0$$

$$u = 2, \text{ or } u = -\sqrt{3}$$

$$\Rightarrow t - 3 = 2 \text{ or } t - 3 = -\sqrt{3}$$

$$\Rightarrow t = 5 \text{ or } t = 3 - \sqrt{3} \text{ (rejected)}$$

$$\Rightarrow x = 25$$

$$\text{Now let } 0 < x < 9,$$

$$-(\sqrt{3} - 2)(t - 3) + (t - 3)^2 - 2\sqrt{3} = 0$$

$$\text{let } t - 3 = u$$

$$u^2 - (\sqrt{3} - 2)u - 2\sqrt{3} = 0$$

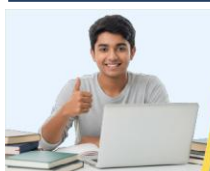
$$\Rightarrow u = \sqrt{3} \text{ or } u = -2$$

$$\Rightarrow t = 3 + \sqrt{3} \text{ (rejected) or } t - 3 = -2$$

$$\Rightarrow t = 1 \Rightarrow x = 1$$

$$\alpha = 1, \beta = 25$$

$$\text{Now } \sqrt{\frac{\beta}{\alpha}} + \sqrt{\alpha\beta} = \sqrt{25} + \sqrt{25} = 10$$



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9. Let $f(x) = \begin{cases} \frac{ax^2 + 2ax + 3}{4x^2 + 4x - 3}, & x \neq -\frac{3}{2}, \frac{1}{2} \\ b, & x = -\frac{3}{2}, \frac{1}{2} \end{cases}$
be continuous at $x = -\frac{3}{2}$. If $f\left(-\frac{3}{2}\right) = \frac{7}{5}$, then x is

equal to :

- (1) 2 (2) 1
(3) 0 (4) 1.4

Ans. (2)

Sol. $f(x) = \begin{cases} \frac{ax^2 + 2ax + 3}{(2x-1)(2x+3)} ; & x \neq -\frac{3}{2}, \frac{1}{2} \\ b ; & x = -\frac{3}{2}, \frac{1}{2} \end{cases}$

for continuous at $x = -\frac{3}{2}$
LHL = RHL

$$\Rightarrow \lim_{x \rightarrow -\frac{3}{2}} \frac{(ax^2 + 2ax + 3)}{(2x-1)(2x+3)}$$

at $x = -\frac{3}{2} \Rightarrow \text{Numerator} = 0$

$$a\left(-\frac{3}{2}\right)^2 + 2a\left(-\frac{3}{2}\right) + 3 = 0$$

$$\frac{9}{4}a - 3a + 3 = 0$$

$$\frac{3a}{4} = 3 \Rightarrow a = 4$$

$$\therefore f(x) = \begin{cases} \frac{4x^2 + 8x + 3}{(2x-1)(2x+3)} ; & x \neq -\frac{3}{2}, \frac{1}{2} \\ b ; & x = -\frac{3}{2}, \frac{1}{2} \end{cases}$$

$$f(x) = \begin{cases} \frac{(2x+1)(2x+3)}{(2x-1)(2x+3)} ; & x \neq -\frac{3}{2}, \frac{1}{2} \\ b ; & x = -\frac{3}{2}, \frac{1}{2} \end{cases}$$

$$f\left(-\frac{3}{2}\right) = \frac{2\left(\frac{2x+1}{2x-1}\right) + 1}{2\left(\frac{2x+1}{2x-1}\right) - 1}$$

$$= \frac{6x+1}{2x+3} = \frac{7}{5}$$

So $x = 1$ Ans.

10. The value of the integral $\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt[3]{\tan 2x}}$ is :

- (1) $\frac{\pi}{12}$ (2) $\frac{\pi}{18}$
(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Ans. (1)

Sol. $I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt[3]{\tan 2x}} \dots (1)$

Apply king

$$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt[3]{\tan 2\left(\frac{\pi}{4} - x\right)}}$$

$$= \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt[3]{\cot 2x}} \dots (2)$$

Add (1) + (2) $\Rightarrow 2I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} (1) dx$

$$I = \frac{1}{2} \left(\frac{\pi}{6} \right) = \frac{\pi}{12}$$

11. Among the statements :

$$I : \text{If } \begin{vmatrix} 1 & \cos \alpha & \cos \beta \\ \cos \alpha & 1 & \cos \gamma \\ \cos \beta & \cos \gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos \alpha & \cos \beta \\ \cos \alpha & 0 & \cos \gamma \\ \cos \beta & \cos \gamma & 0 \end{vmatrix}$$

, then $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{3}{2}$, and

$$II : \text{If } \begin{vmatrix} x^2 + x & x+1 & x-2 \\ 2x^2 + 3x - 1 & 3x & 3x-3 \\ x^2 + 2x + 3 & 2x-1 & 2x-1 \end{vmatrix} = px + q, \text{ then}$$

$$p^2 = 196q^2,$$

- (1) both are false (2) only II is true
(3) both are true (4) only I is true

Ans. (1)



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Sol. Let $\cos \alpha = x$

$$\cos \beta = y$$

$$\cos \gamma = z$$

$$\begin{vmatrix} 0 & x & y \\ x & 0 & z \\ y & z & 0 \end{vmatrix} = \begin{vmatrix} 1 & x & y \\ x & 1 & z \\ y & z & 1 \end{vmatrix}$$

Expanding both sides, we get

$$x^2 + y^2 + z^2 = 1$$

$$\text{i.e. } \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

Statement 1 is false

Now,

$$\begin{vmatrix} x^2 + x & 1 + x & x - 2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = px + q$$

Put $x = 0$ both sides

$$q = \begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 3 & -1 & -1 \end{vmatrix}$$

$$\Rightarrow q = -12$$

Now put $x = 1$ both sides

$$p + q = \begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 3 \\ 6 & 1 & 1 \end{vmatrix} = 42$$

$$\Rightarrow p = 54$$

$$\text{Now } \frac{p^2}{q^2} = \left(\frac{54}{-12} \right)^2 + 196$$

$$\Rightarrow p^2 \neq 196q^2$$

Statement (2) is false

Correct option (1)

12. The value of $\frac{{}^{100}C_{50}}{51} + \frac{{}^{100}C_{51}}{52} + \dots + \frac{{}^{100}C_{100}}{101}$ is :

$$(1) \frac{2^{101}}{100} \quad (2) \frac{2^{100}}{100}$$

$$(3) \frac{2^{101}}{101} \quad (4) \frac{2^{100}}{101}$$

Ans. (4)

$$\text{Sol. } S = \sum_{r=50}^{100} \frac{{}^{100}C_r}{r+1} = \sum_{r=50}^{100} \frac{1}{r+1} \cdot \frac{r+1}{101} \cdot {}^{101}C_{r+1}$$

$$S = \frac{1}{101} \sum_{r=50}^{100} {}^{101}C_{r+1}$$

$$= \frac{1}{101} \times \frac{2^{101}}{2} = \frac{2^{100}}{101}$$

Option (4)

13. Let the mean and variance of 8 numbers $-10, -7, -1, x, y, 9, 2, 16$ be $\frac{7}{2}$ and $\frac{293}{4}$, respectively. Then the mean of 4 numbers $x, y, x+y+1, |x-y|$ is:

$$(1) 11 \quad (2) 9$$

$$(3) 10 \quad (4) 12$$

Ans. (1)

$$\text{Sol. Mean} = \frac{-18 + x + y + 2 + 9 + 16}{8} = \frac{7}{2}$$

$$= \frac{x + y + 9}{8} = \frac{7}{2} \Rightarrow x + y + 9 = 28 \quad \dots (1)$$

$$\text{Variance} = \frac{\sum z_i^2}{8} - (\mu)^2 = \frac{293}{4}$$

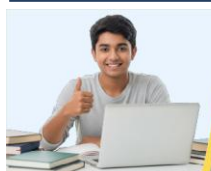
$$\Rightarrow \frac{10^2 + 7^2 + 1^2 + x^2 + y^2 + 2^2 + 9^2 + 16^2}{8} - \left(\frac{7}{2} \right)^2 = \frac{293}{4} \quad \dots (2)$$

$$\text{Solving (1) \& (2)} \Rightarrow x = 12, y = 7$$

Mean of $(1+x+y), x, y, |y-x|$ is

$$\Rightarrow \frac{20 + 12 + 7 + 5}{4} = \frac{44}{4} = 11$$

Option (1)



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14. The sum of all possible values of $n \in \mathbb{N}$, so that the coefficients of x , x^2 and x^3 in the expansion of $(1+x^2)^2(1+x)^n$, are in arithmetic progression is :

- (1) 3 (2) 7
(3) 12 (4) 9

Ans. (4)

Sol. $(x^4 + 2x^2 + 1)({}^nC_0x^0 + {}^nC_1x^1 + {}^nC_2x^2 + {}^nC_3x^3 + \dots)$

Coefficient $x \Rightarrow {}^nC_1$,

coeff. of $x^2 \Rightarrow 2 + {}^nC_2$

$$2 + \frac{n(n-1)}{2}$$

Coeff. of $x^3 = 2 \cdot {}^nC_1 + {}^nC_3$

$$= 2n + \frac{n(n-1)(n-2)}{6} \quad (\text{if } n \geq 3)$$

Now according to question

$$n + 2n + \frac{n(n-1)(n-2)}{6} = 2 \left[2 + \frac{n(n-1)}{2} \right]$$

$$3n + \frac{n(n-1)(n-2)}{6} = 4 + n(n-1)$$

$$\Rightarrow n^3 - 9n^2 + 26n - 24 = 0$$

$$\Rightarrow n = 2, 3, 4 \quad \Rightarrow n = 3, 4$$

Now checking for $n = 2$

$$\left. \begin{array}{l} \text{Coeff. of } x=2 \\ \text{Coeff. of } x^2=3 \\ \text{Coeff. of } x^3=4 \end{array} \right\} \Rightarrow \text{are in A.P.}$$

$\Rightarrow n = 2$ is also the correct choice

Required sum of values of 'n'

$$= 2 + 3 + 4 = 9$$

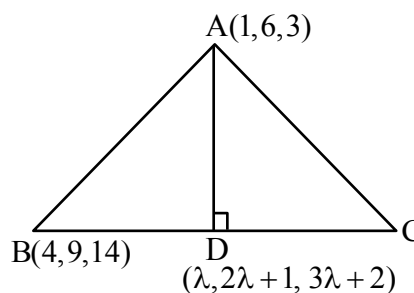
Option (4)

15. The vertices B and C of a triangle ABC lie on the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{z-2}{3}$. The coordinates of A and B are (1, 6, 3) and (4, 9, α) respectively and C is at a distance of 10 units from B. The area (in sq. units) of ΔABC is:

- (1) $5\sqrt{13}$ (2) $15\sqrt{13}$
(3) $20\sqrt{13}$ (4) $10\sqrt{13}$

Ans. (1)

Sol. $\frac{4}{1} = \frac{9-1}{2} = \frac{\alpha-2}{3} \Rightarrow \alpha = 14$



$$\overrightarrow{AD} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 0$$

$$(\lambda-1)\hat{i} + (2\lambda-5)\hat{j} + (3\lambda-1)\hat{k} = \overrightarrow{AD}$$

$$\Rightarrow \lambda-1 + 4\lambda-10 + 9\lambda-3 = 0$$

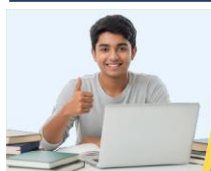
$$\Rightarrow 14\lambda = 14 \Rightarrow \lambda = 1$$

$$D = (1, 3, 5)$$

$$AD = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$\text{Ar}(\Delta ABC) = \frac{1}{2} \times \sqrt{13} \times 10 = 5\sqrt{13}$$

Option (A)



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16. Number of solutions of

$$\sqrt{3} \cos 2\theta + 8 \cos \theta + 3\sqrt{3} = 0, \theta \in [-3\pi, 2\pi] \text{ is:}$$

- (1) 0 (2) 5
(3) 3 (4) 4

Ans. (2)

Sol. $\sqrt{3} (2 \cos^2 \theta - 1) + 8 \cos \theta + 3\sqrt{3} = 0$

$$2\sqrt{3} \cos^2 \theta + 8 \cos \theta + 2\sqrt{3} = 0$$

$$(\sqrt{3} \cos \theta + 1)(\cos \theta + \sqrt{3}) = 0$$

$$\cos \theta = -\frac{1}{\sqrt{3}}$$

as $-\sqrt{3}$ (reject)

$\therefore \theta =$ will have 5 value in $[-3\pi, 2\pi]$

Ans. = 5 $\Rightarrow$ option (2)

17. Let $S = \{z : 3 \leq |2z - 3(1 + i)| \leq 7\}$ be a set of

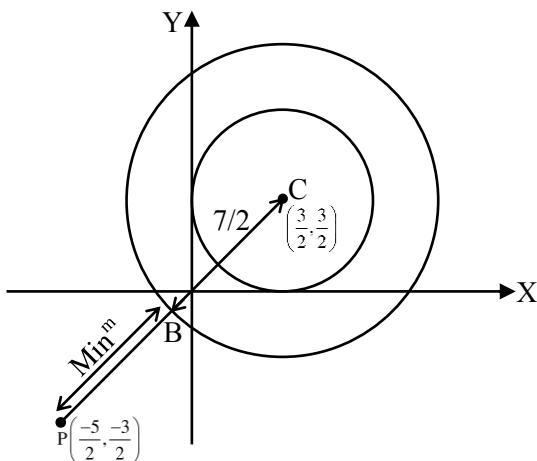
complex numbers. Then $\min_{z \in S} \left| z + \frac{1}{2}(5 + 3i) \right|$ is

equal to :

- (1) $\frac{1}{2}$ (2) $\frac{3}{2}$
(3) 2 (4) $\frac{5}{2}$

Ans. (2)

Sol. $\frac{3}{2} \leq \left| z - \frac{3}{2}(1 + i) \right| \leq \frac{7}{2}$



$$\min_{z \in S} \left| z - \left(\frac{-5}{2} - \frac{3}{2}i \right) \right| = PB$$

$$PB = PC - \frac{7}{2} \Rightarrow 5 - \frac{7}{2} \Rightarrow \frac{3}{2}$$

Option (2)

18. Let α and β respectively be the maximum and the minimum values of the function

$$f(\theta) = 4 \left(\sin^4 \left(\frac{7\pi}{2} - \theta \right) + \sin^4 (11\pi + \theta) \right)$$

$$-2 \left(\sin^6 \left(\frac{3\pi}{2} - \theta \right) + \sin^6 (9\pi - \theta) \right), \theta \in \mathbf{R}.$$

Then $\alpha + 2\beta$ is equal to :

- (1) 4 (2) 5
(3) 3 (4) 6

Ans. (2)

Sol. $f(\theta) = 4 \left(\sin^4 \left(\frac{7\pi}{2} - \theta \right) + \sin^4 (11\pi + \theta) \right) - 2 \left(\sin^6 \left(\frac{3\pi}{2} - \theta \right) + \sin^6 (9\pi - \theta) \right)$

$$f(\theta) = 4(\cos^4(\theta) + \sin^4(\theta)) - 2(\cos^6\theta + \sin^6\theta)$$

$$f(\theta) = 4(1 - 2\sin^2\theta\cos^2\theta) - 2(1 - 3\sin^2\theta\cos^2\theta)$$

$$f(\theta) = 2 - 2\sin^2\theta\cos^2\theta$$

$$f(\theta) = 2 - \frac{\sin^2(2\theta)}{2}$$

$$\alpha = f(\theta)_{\max} = 2$$

$$\beta = f(\theta)_{\min} = \frac{3}{2}$$

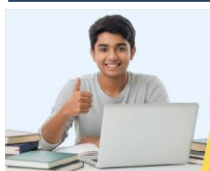
$$\Rightarrow \alpha + 2\beta = 5$$

Ans. = 5 option (2)

19. A building construction work can be completed by two masons A and B together in 22.5 days. Mason A alone can complete the construction work in 24 days less than mason B alone. Then mason A alone will complete the construction work in:

- (1) 24 days (2) 42 days
(3) 30 days (4) 36 days

Ans. (4)



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Sol. Let time taken by mason a alone to complete the work be in x days so, mason B along take $x + 24$ days

$$\text{work done by A in 1 day} = \frac{1}{x}$$

$$\text{work done by B in 1 day} = \frac{1}{x + 24}$$

$$\text{so work done by A + B in 1 day} = \frac{1}{22.5}$$

$$\text{So, } \frac{1}{x} + \frac{1}{x + 24} = \frac{2}{45}$$

$$x^2 - 21x - 540 = 0$$

$$x = 36 \text{ \&}$$

$$x = -15 \text{ (rejected)}$$

$$\text{Ans.} = 36 \text{ option (4)}$$

20. Let the direction cosines of two lines satisfy the equations : $4\ell + m - n = 0$ and $2mn + 10n\ell + 3\ell m = 0$.

Then the cosine of the acute angle between these lines is :

$$(1) \frac{10}{\sqrt{38}}$$

$$(2) \frac{20}{3\sqrt{38}}$$

$$(3) \frac{10}{7\sqrt{38}}$$

$$(4) \frac{10}{3\sqrt{38}}$$

Ans. (4)

Sol. Direction cosines of two lines satisfy the equation

$$\Rightarrow 4\ell + m - n = 0 \quad \dots(1)$$

$$2mn + 10n\ell + 3\ell m = 0 \quad \dots(2)$$

& we know

$$\Rightarrow \ell^2 + m^2 - n^2 = 1 \quad \dots(3)$$

$$\Rightarrow n = 4\ell + m \text{ putting in eqn. (1)}$$

$$\Rightarrow n(2m + 10\ell) + 3\ell m = 0$$

$$\Rightarrow (4\ell + m)(2m + 10\ell) + 3\ell m = 0$$

$$\Rightarrow 8\ell m + 40\ell^2 + 2m^2 + 10\ell m + 3\ell m = 0$$

$$\Rightarrow 40\ell^2 + 21\ell m + 2m^2 = 0$$

$$\Rightarrow (8\ell + m)(5\ell + 2m) = 0$$

$$\text{Case 1 : } 8\ell + m = 0 \Rightarrow m = -8\ell$$

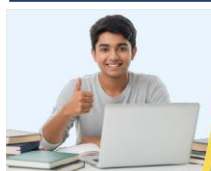
$$\text{Case 2 : } 5\ell + 2m = 0 \Rightarrow m = \frac{-5}{2}\ell$$

So direction ratio of L_1 is $\ell, -8\ell, -4\ell$

& direction ratio of L_2 is $\ell, \frac{-5\ell}{2}, \frac{3\ell}{2}$

$$\begin{aligned} \cos \theta &= \left| \frac{\ell^2 + 20\ell^2 - 6\ell^2}{\sqrt{\ell^2 + 64\ell^2 + 16\ell^2} \sqrt{\ell^2 + \frac{25\ell^2}{4} + \frac{9\ell^2}{4}}} \right| \\ &= \frac{15\ell^2}{(9\ell) \frac{\sqrt{38}\ell}{2}} = \frac{10}{3\sqrt{38}} \end{aligned}$$

$$\text{Ans.} = \frac{10}{3\sqrt{38}}$$



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SECTION-B

21. Let $|A| = 6$, where A is a 3×3 matrix. If $|\text{adj}(3\text{adj}(A^2 \cdot \text{adj}(2A)))| = 2^m \cdot 3^n$, $m, n \in \mathbb{N}$, then $m + n$ is equal to _____.

Ans. (62)

Sol. $\text{adj } 2A = 2^2 \text{ adj } A \quad \because \text{adj } kA = k^{n-1} (\text{adj } A)$

$$= 4 \text{ adj } A$$

$$\text{Now } A^2 (\text{adj } 2A) = 4A (\text{adj } A)$$

$$= 4A |A| I_3$$

$$= 24A$$

$$\text{Now } 3 \text{ adj } (A^2 (\text{adj } 2A)) = 3 \text{ adj } (24A)$$

$$= 3 \cdot (24)^2 \text{ adj } A$$

$$\text{Now } \left| \text{adj} \left(3 \text{ adj } (A^2 (\text{adj } 2A)) \right) \right|$$

$$= \left| \text{adj} \left(3 \cdot (24)^2 \text{ adj } A \right) \right|$$

$$= \left| \left(3 \cdot (24)^2 \right)^2 (\text{adj adj } A) \right|$$

$$= \left| 3^6 \cdot 2^{12} \text{ adj adj } A \right|$$

$$= (3^6 \cdot 2^{12})^3 |\text{adj adj } A|$$

$$= 3^{18} \cdot 2^{36} \cdot (A)^4$$

$$3^{22} \cdot 2^{40}$$

$$\therefore m + n = 62$$

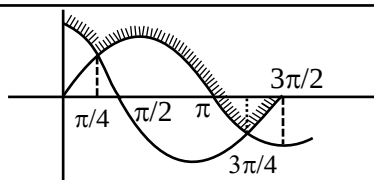
22. Let the area of the region bounded by the curve

$$y = \max \{ \sin x, \cos x \}, \text{ lines } x = 0, x = \frac{3\pi}{2}, \text{ and}$$

the x-axis be A . Then, $A + A^2$ is equal to

Ans. (12)

Sol.



$$A = \int_0^{\pi/4} \cos x \, dx + \int_{\pi/4}^{\pi} \sin x \, dx + \int_{\pi}^{5\pi/4} -\sin x \, dx + \int_{5\pi/4}^{3\pi/2} -\cos x \, dx$$

$$A = (\sin x)_0^{\pi/4} + (\cos x)_{\pi/4}^{\pi} + (\cos x)_{\pi}^{5\pi/4} + (\sin x)_{5\pi/4}^{3\pi/2}$$

$$A = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 1 + 1 - \frac{1}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}} = 3$$

$$A^2 + A = 12$$

23. Let f be a twice differentiable non-negative function such that

$$(f(x))^2 = 25 + \int_0^x \left((f(t))^2 + (f'(t))^2 \right) dt. \text{ Then the}$$

mean of $f(\log_e(1))$, $f(\log_e(2))$, ..., $f(\log_e(625))$ is equal to _____.

Ans. (1565)

Sol. $2f(x)f'(x) = f^2(x) + (f'(x))^2$

$$\Rightarrow (f(x) - f'(x))^2 = 0$$

$$\Rightarrow f(x) = f'(x)$$

$$\Rightarrow \ln(f(x)) = x + c \Rightarrow f(x) = c'e^x$$

$$f(0) = 5 \Rightarrow f(x) = 5e^x$$

$$\text{Mean} = \frac{f(\ln 1) + f(\ln 2) + \dots + f(\ln 625)}{625}$$

$$= \frac{5[1 + 2 + \dots + 625]}{625} = 1565$$



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24. From the first 100 natural numbers, two numbers first a and then b are selected randomly without replacement. If the probability that $a - b \geq 10$ is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to _____.

Ans. (311)

Sol. $a - b \geq 10$

$$\text{Total cases} = 100 \times 99$$

$$\text{Fav. Cases} = 1 + 2 + 3 + \dots + 90$$

$$\text{Req. Prob} = \frac{1 + 2 + \dots + 90}{100 \times 99}$$

$$\frac{m}{n} = \frac{90 \left(\frac{91}{2} \right)}{100(99)} = \frac{91}{220}$$

$$m + n = 311$$

25. The number of 4-letter words, with or without meaning, which can be formed using the letters PQRPRSTUVP, is _____.

Ans. (1422)

Sol. $P \rightarrow 3, Q \rightarrow 2, R \rightarrow 2, S, T, U, V$

Case I 3 alike, 1 different

$${}^1C_1 \times {}^6C_1 \times \frac{4!}{3!} = 24$$

Case II 2 alike, 2 alike

$${}^3C_2 \times \frac{4!}{2!2!} = 18$$

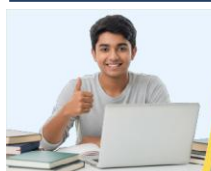
Case III 2 alike, 2 different

$${}^3C_1 \times {}^6C_2 \times \frac{4!}{2!} = 540$$

Case IV All 4 different

$${}^7C_4 \times 4! = 840$$

$$\text{Total words} = 1422$$



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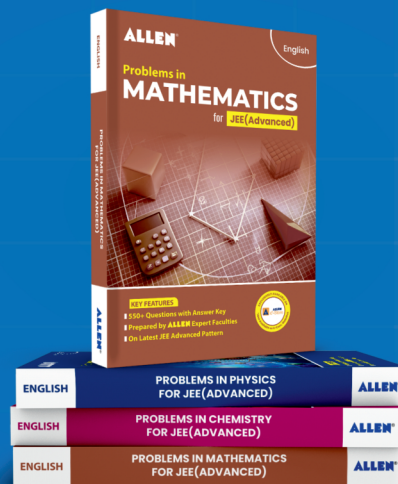
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